Evaluation of polynomial regression models for the Student t and Fisher F critical values, the best interpolation equations from double and triple natural logarithm transformation of degrees of freedom up to 1000, and their applications to quality control in science and engineering

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ABSTRACT

Serious gaps exist in the present critical value tables for the Student t and Fisher F or ANOVA significance tests. Statistically correct applications of these tests to the experimental data therefore become difficult. A total of 18 different regression models were evaluated for the Student t and 24 for the Fisher F critical values. These models varied from simple polynomial (quadratic to 7th order) to the combined single (ln), double (lnln), or triple (lnlnln) natural-logarithm- (ln-) transformed polynomial models. The advantage of ln-, lnln- or lnlnln-transformations of the degrees of freedom for interpolating the Student t and Fisher F critical values has been documented for the first time in the published literature. The use of critical value equations applicable in the range of 1–1000 degrees of freedom for ln-transformation, 2–1000 for lnln-transformation, or 3–1000 for lnlnln-transformation, instead of the tables, is proposed as a 21st century innovation for the computer programming of these significance tests. A number of application examples are pointed out to illustrate the usefulness of this work.

Key words: F-ratio, ANOVA, critical value, degrees of freedom, reference material, significance tests.

RESUMEN

Las tablas de valores críticos para las pruebas de significado de t de Student y F de Fisher o ANOVA, se caracterizan por serias deficiencias. Aplicaciones estadísticamente correctas de estas pruebas a los datos experimentales, por lo tanto, se hacen difíciles. Se evaluaron un total de 18 modelos de regresión para los valores críticos de t de Student y 24 modelos para los de F de Fisher. Estos modelos varían de modelos polinomiales (de tipo cuadrático hasta la 7^{a}. potencia) sencillos hasta los polinomiales combinados con la transformación logarítmica natural (ln) de tipo sencillo (ln), doble (lnln), o triple (lnlnln). Las ventajas de la transformaciones ln, lnln o lnlnln de los grados de libertad para la interpolación de los valores *críticos de t de Student y F de Fisher se demuestran por primera vez en la literatura publicada. Para la programación computacional de dichas pruebas de significado se propone, en lugar de las tablas, como la innovación del siglo XXI, el uso de las ecuaciones de valores críticos aplicables en el intervalo de 1–1000 grados de libertad para la transformación ln, de 2–1000 para la transformación lnln, o de 3–1000 para la transformación lnlnln. Se presentan una serie de ejemplos de aplicación con el fin de ilustrar la utilidad de este trabajo.*

Palabras clave: Relación F, ANOVA, valor crítico, grados de libertad, materiales de referencia, pruebas de significado o significancia.

INTRODUCTION

Quality control in all branches of science and engineering demands the application of significance tests, such as the Student t, Fisher F or F-ratio, and ANOVA or analysis of variance (*e.g.*, Anderson, 1987; Ebdon, 1988; Otto, 1999; Jensen *et al.*, 2000; Miller and Miller, 2000; Bevington and Robinson, 2003; Verma, 2005; Walker and Maddan, 2005). It is customary to apply these tests at a given predetermined confidence level (CL) such as 95% (*e.g.*, Miller and Miller, 2000) or 99% (*e.g.*, Verma, 1998, 2005; Verma and Quiroz-Ruiz, 2008). Such critical values or percentage points should thus be available for *all* degrees of freedom (dF or v) required for their statistically correct application. An examination of the published literature readily reveals that this is not the case.

The critical values for the Student t test are available in most literature sources (*e.g.*, Verma, 2005) as a total of 42 critical values corresponding to v = 1(1)30(5)50(10)100 (100)200(300)500(500)1000 (each set being for seven two-sided CL of 60% to 99.8%, or equivalently one-sided CL of 80% to 99.9%), and for the Fisher F test as 20 values of horizontal dF (HdF) v_1=1(1)12(3)15(5)30(10)50 (50)100(900)1000 and 39 values of vertical dF (VdF) v_2= 1(1)30(5)40(10)60(20)100(100)200(300)500(500)1000 (the Fisher F values are generally available for 95% and 99% CL). This shows that serious gaps exist in the critical value tables for these very frequently used significance tests, *e.g.*, within the dF range of 1-1000 and for any given CL, 958 values out of 1000 are missing for the Student t and a total of 980×961 values for the Fisher F. Note that the critical value corresponding to the dF of ∞ is not considered here because the ∞ is an undefined number in mathematical terms and refers to the population (and not to a statistical sample).

I present a new methodology for the interpolation of the existing critical values, evaluate 18 and 24 different regression models for the Student t and Fisher F, respectively, and propose the new best-fitted polynomial double or triple natural logarithm-transformed equations (defined as lnln and lnlnln functions, respectively) that allow us to extend the availability of critical values for all dF (v for the Student t; v_1 and v_2 for the Fisher F) from 1 up to 1000, *i.e.*, 1(1)1000.

REGRESSION PROCEDURE AND THE INTERPOLATION OF CRITICAL VALUES

For the manipulation of critical values, linear to cubic regressions have been used in the literature (*e.g.*, Bugner and Rutledge, 1990; Rorabacher, 1991; Verma *et al.*, 1998). In fact, I tried several polynomial fits (from quadratic up to 7th order) to obtain new equations for the Student t and Fisher F critical values, but to my surprise none of them performed satisfactorily for interpolation purposes (see Figure 1 and the explanation below in this section). The failure of the polynomial fits motivated me to perform some kind of data transformation before undertaking the polynomial regressions. Single natural-logarithm (ln) transformation for statistically correct handling of compositional data was proposed long ago by Aitchison (1986) and was used a couple of years ago by Verma *et al.* (2006) for proposing new discriminant function diagrams. To my pleasant surprise, this methodology has also recently provided excellent interpolations of critical values of discordancy tests (Verma and Quiroz-Ruiz, 2008).

Although representing an incomplete treatment of compositional data, log-transformation has been traditionally used for fitting a linear function to a log-transformed compositional ratio variable (Na/K) in geothermal fluid geothermometry (Fournier, 1979; Verma and Santoyo, 1997; Díaz-González *et al.*, 2008). For this example of geothermics to be comparable to my present work, quadratic and higher-order regression fits should have been evaluated. Furthermore, a correct log-ratio transformation would be to use more than two compositional variables and a common denominator for log-ratios as suggested by Aitchison (1986, 1989), Verma *et al.* (2006), and Agrawal *et al.* (2008); dealing with just one such ratio (Na/K) is not sufficient to recognize the multivariate nature of the compositional data (Aitchison, 1989; Agrawal and Verma, 2007).

Prior to the polynomial regressions, three types of natural-logarithm transformations of the dF (v, v_1, and v_2) –called here as the ln, lnln, and lnlnln functions– were carried out and evaluated for the first time in the published literature. These three ln-transformations mean that one uses the ln(v), ln(ln(v)), and ln(ln(ln(v))) variables, respectively, instead of the raw v for the Student t, or the raw v_1 or v_2

for the Fisher F, in the theoretical regression function. The results of the evaluation of the quadratic to 7th order fits are graphically presented in Figure 1.

First, the quality parameter R^2 called the multiple-correlation coefficient (Bevington and Robinson, 2003) was used (Figures 1a, 1c, 1e). R^2 is simply an extrapolation of the well-known concept of the linear-correlation coefficient *r*, which characterizes the correlation between two variables at a time, to include multiple correlations, such as polynomial correlations, between groups of variables taken simultaneously. The parameter *r* is useful for testing whether one particular variable should be included in the theoretical function that is fitted to the data, whereas the parameter R^2 characterizes the fit of the data to the entire function (Bevington and Robinson, 2003; Verma and Quiroz-Ruiz, 2008; Verma *et al.*, 2009). Thus, a comparison of the R^2 for different functions is useful in optimizing the theoretical functional forms such as those evaluated in the present work (Figures 1a, 1c, 1e).

Secondly, the sum of the squared residuals $SSR = \{\sum(cv_{\text{table}}\text{-}cv_{\text{calc}})^2\}_{\text{int}}$ was investigated as the other quality parameter (Figures 1b, 1d, 1f), where cv_{table} is the value listed in a table for the Student t or Fisher F test for any given CL, and cv_{calc} is the value calculated from the corresponding regression equation; the subscript $_{\text{int}}$ emphasizes that the regression equations are for the interpolation purposes only, and should not be generally used for the extrapolation of the data, although Verma and Quiroz-Ruiz (2008) have shown that such ln-transformed equations may as well be useful for the extrapolation purpose. No attempt was made to normalize this quality parameter (*SSR*) with respect to the number of tabulated critical values for a given case, nor with respect to some other variable such as the mean critical value, because the main interest was to use it for the visual comparison of the different (18 for the Student t and 24 for the Fisher F) regression models (Figures 1b, 1d, 1f). Nevertheless, the use of normalized *SSR* values will only change the vertical scale in Figure 1, without any modification in the observed trend.

It is readily seen that in all cases for 99% CL (Figure 1), the R^2 parameter for purely polynomial fits from the quadratic (q) to the 7th order (p7) is consistently small (0.08044–0.44374 in Figure 1a; 0.36301–0.55106 in Figure 1c; and 0.37629–0.59237 in Figure 1e) and the corresponding SSR parameter is unduly large (60–34 in Figure 1b; 10,500–8,600 in Figure 1d; and 5.7–4.0 in Figure 1f) to be of any use in such interpolations. The improvement from any of the ln-, lnln-, and lnlnln-transformations preceding the polynomial fits of the q to 7th order is highly significant because the fitting quality parameter R^2 varied, respectively, from 0.71096 to 0.99845, 0.97063 to 1.00000, and not reported (because lnln-transformation already reaches the theoretical maximum value of 1) in Figure 1a; from 0.94694 to 0.99761, 0.97097 to 0.99424, and 0.93975 to 0.98627 in Figure 1c, and from 0.95395 to 1.00000, 0.9380 to 0.99999, and 0.99365 to 0.99999 in Figure 1e. When for a polynomial model involving ln-transformation the R^2 practically approaches the theoretical maximum value of 1, any further improvement in the fitting-quality is impossible to attain even if one uses a higher-order polynomial or a more complex ln-transformation. The *SSR* parameter for the ln-transformed models correspondingly is extremely small (17.7–0.1, 1.8–(2.8×10^{-6}), and not reported in Figure 1a; 2,100–150, 700–150, and 880–200 in Figure 1d; and 0.9–9×10^{-5}, 0.07–8×10^{-5}, and 0.05–8×10^{-5} in Figure 1f) as compared to the respective simpler polynomial models (see above). The relatively large squared residuals in Figure 1d as compared to those in Figure 1f are due to the fact that the 99% CL critical values for the Fisher F corresponding to VdF=1 and HdF=1–1,000 are much greater (405–637) than those for VdF=1,000 and HdF=1–1,000 (6.66–1.16; for critical values see any standard textbook on the subject; *e.g.*, Anderson, 1987; Urbina-Medal and Valencia-Ramírez, 1987; Verma, 2005).

Similarly, also for purely polynomial fits for the Student t and Fisher F tests corresponding to 95% CL critical values (plots not shown) the R^2 parameter was consistently very small (0.1070–0.5272 and 0.0180–0.5053, respectively) and the *SSR* parameter was consistently unreasonably large (about 840–6 and 19,900–1,000, respectively). At this CL (95%) the ln-, lnln-, and lnlnln-transformations prior to the polynomial fits provided much greater R^2 values, respectively, from 0.4937–1.0000, 0.9659–1.0000, and 0.9768–1.0000 (for HdF=1 and VdF=1–1000, or VdF=1 and HdF=1–1000). The *SSR* parameter correspondingly was extremely small for the ln-, lnln-, and lnlnln- transformed models (as low as 0.024, 0.008, and 0.021, respectively).

These kinds of results and trends were shown by all other critical value sets as well, *i.e.*, the superiority of the ln-transformed polynomial models as compared to the simpler polynomial models (without ln-transformation) has been demonstrated beyond any doubt.

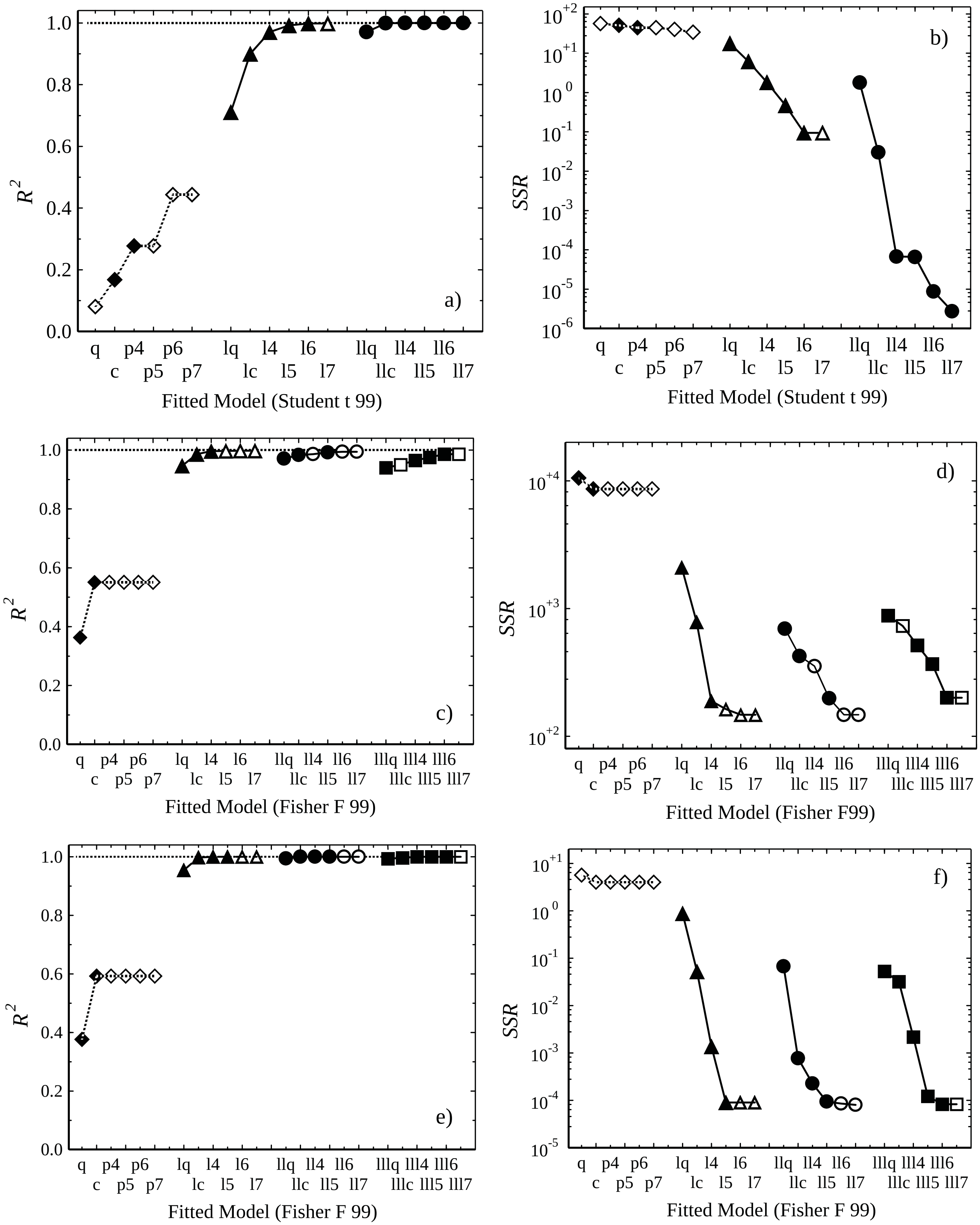
BEST-FIT EQUATIONS

I decided to call the best-fit equation as the one that: (i) provided R^2 close to 1 (in fact, practically equal to 1); (ii) showed small sum of absolute ($\text{SAR}=\sum Abs(cv_{\text{calc}}\text{-}cv_{\text{table}})$), or squared (*SSR* defined above) residuals; and (iii) was based on the smallest number of regression terms and the less complex ln-transformation, *i.e.*, under similar circumstances, the ln function was preferred as compared to the lnln function and the latter as compared to the lnlnln function.

For the Student t critical values, the lnln-transformed 5th order best-fit equation for v=2–1000 is:

$$cv_{\text{calc}} = \mathrm{I} + F_1\cdot(\ln(\ln(v)) + F_2\cdot(\ln(\ln(v))^2 + F_3\cdot(\ln(\ln(v))^3 + F_4\cdot(\ln(\ln(v))^4 + F_5\cdot(\ln(\ln(v))^5 \qquad (1)$$

In equation (1) of the 5th order polynomial regression involving lnln-transformation, *I* is the intercept term and F_1,


a)
b)
c)
d)
e)
f)
R^2
SSR
Fitted Model (Student t 99)
Fitted Model (Fisher F 99)
Fitted Model (Fisher F99)
q c p4 p5 p6 p7 lq lc l4 l5 l6 l7 llq llc ll4 ll5 ll6 ll7
q c p4 p5 p6 p7 lq lc l4 l5 l6 l7 llq llc ll4 ll5 ll6 ll7 lllq lllc lll4 lll5 lll6 lll7

F_2, F_3, F_4, and F_5 are the coefficients of the linear, quadratic, 3rd, 4th, and 5th order terms, respectively. All these coefficients of the best-fit equation (1) have been summarized in Table 1. This equation can be used to compute any critical value for v=2–1000 and for any desired CL (see Table 1).

For the Fisher F tables, two different sets of best-fit equations had to be proposed for any given CL, one for the interpolation of the vertical dF (VdF v_2) for a given horizontal dF (HdF v_1), and the other for the HdF (v_1) for a given VdF (v_2) and CL. For example, the following double ln-transformed 6th order best-fit equation applicable for v_2= 2–1000 for a given v_1 is:

$$cv_{\text{calc}} = I + F_1\cdot(\ln(\ln(v_2))) + F_2\cdot(\ln(\ln(v_2)))^2 + F_3\cdot(\ln(\ln(v_2)))^3 + F_4\cdot(\ln(\ln(v_2)))^4 + F_5\cdot(\ln(\ln(v_2)))^5 \quad (2)$$

Similarly, the following double ln-transformed 5th order best-fit equation applicable for v_1= 2–1000 for a given v_2 is:

$$cv_{\text{calc}} = I + F_1\cdot(\ln(\ln(v_1))) + F_2\cdot(\ln(\ln(v_1)))^2 + F_3\cdot(\ln(\ln(v_1)))^3 + F_4\cdot(\ln(\ln(v_1)))^4 + F_5\cdot(\ln(\ln(v_1)))^5 \quad (3)$$

The values of the coefficients for the 99% CL are listed in Tables 2 and 3, respectively.

Finally, not all combinations of HdF and VdF are covered by equations 2 and 3. For example, critical values will be missing for the combination of v_1=13 and v_2=31–34, 36–39, 41–49, 51–59, 61–79, 81–99, 101–199, 201–499, and 501–999. First, I tried to evaluate more complex polynomial models involving different kinds of ln-transformations of simultaneously both v_1 and v_2 in a single equation, but failed to obtain any acceptable solution.

Therefore, a "second-round" of equations had to be proposed to complete the missing Fisher F values. As an example, the following triple ln-transformed 6th order best-fit "second-round" equation applicable for v_2= 3–1000 for a given v_1 is:

$$cv_{\text{calc}} = I + F_1\cdot(\ln(\ln(\ln(v_2)))) + F_2\cdot(\ln(\ln(\ln(v_2))))^2 + F_3\cdot(\ln(\ln(\ln(v_2))))^3 + F_4\cdot(\ln(\ln(\ln(v_2))))^4 + F_5\cdot(\ln(\ln(\ln(v_2))))^5 + F_6\cdot(\ln(\ln(\ln(v_2))))^6 \quad (4)$$

The values of the coefficients for the 99% CL are listed in Table 4 for v_1=13–29, thus, completing the Fisher F critical values for v_1=1(1)30. Because for the F-ratio and ANOVA tests, v_1 (HdF) refers to the dF that correspond to the number of classes or groups, the above equations and Tables 2-4 cover most, if not all, applications of these significance tests. However, if there were still needs for greater values of v_1, the present method of the triple ln-transformed 6th order fit can be easily extended to include any missing cases.

Another example includes the following double ln-transformed 5th order best-fit "second-round" equation applicable for v_1= 2–1000 for a given v_2 (note equation 5 is identical to equation 3):

$$cv_{\text{calc}} = I + F_1\cdot(\ln(\ln(v_1))) + F_2\cdot(\ln(\ln(v_1)))^2 + F_3\cdot(\ln(\ln(v_1)))^3 + F_4\cdot(\ln(\ln(v_1)))^4 + F_5\cdot(\ln(\ln(v_1)))^5 \quad (5)$$

The values of the coefficients for the 99% CL (Table 5) are for v_2=31–49. They complete the Fisher F critical values for v_2=1(1)50.

As stated earlier, this new methodology involving lnln- or lnlnln-transformations can be easily extended to calculate any other critical value for the F-ratio or ANOVA test. Similar equations and Tables were also generated for the Fisher F 95% CL, but are not included here; these are available by email request to the author.

This methodology of ln-, lnln-, or lnlnln-transformation should be useful to handle all other types of critical value tables if one is interested in precisely estimating interpolated values for their application in significance tests (this work), discordancy tests (Verma and Quiroz-Ruiz, 2008), or any other type of statistical tests. On the other hand, because the best interpolation equations based on these innovative natural logarithm-transformations along with polynomial fits provide ideal solutions with R^2 values of practically 1 (maximum theoretical value attainable) and extremely small *SSR* values, I consider no need to try any other conventional fitting methods such as numerical computational methods or artificial neural network (ANN) methodology (Verma *et al.*, 2008; Díaz-González *et al.*, 2008).

It will be a good idea to abandon the use of the critical value tables; instead, the new critical value equations can be easily programmed in spreadsheets as well as in new computer software. Thus, the use of critical value equa-

Figure 1. Evaluation of the quality of 18 different polynomial regression models for the interpolation of critical values for the Student t test and of 24 models for the Fisher F test, through two parameters R^2 and *SSR*. The terms q, c, p4, p5, p6, and p7 are for the simple (without ln-transformation) polynomial regressions using quadratic, cubic, 4th order, 5th order, 6th order, and 7th order, respectively; the prefixes "l", "ll", and "lll" before these terms signify, respectively, the ln-, lnln-, and lnlnln-transformation models, respectively, of the dF. The lnlnln-transformations were also tried for the Student t test, but are not shown here because they did not represent any significant improvement over the lnln-transformations, which were already excellent. The symbols used for the simple (without any ln-transformation) and ln-, lnln- and lnlnln-transformed polynomial models are diamonds, triangles, circles, and squares, respectively (the filled symbols are used for a model for which most, if not all, terms in a model were significant at the 99% confidence level (CL) whereas the open symbols are when two or more terms were not significant, or the final highest order term was zeroed). Fitted models are shown as: (a) R^2 for the Student t test for two-tailed 99% CL; (b) *SSR* for the Student t test for two-tailed 99% CL; (c) R^2 for the Fisher F test for 99% CL with VdF=1 and HdF=1-1,000; (d) *SSR* for the Fisher F test for 99% CL with the same set as in (c); (e) R^2 for the Fisher F test for 99% CL with VdF=1,000 and HdF=1-1,000; and (f) *SSR* for the Fisher F test for 99% CL with the same set as in (e).

Table 1. The best double natural-logarithm transformed regression model for the interpolation of Student t critical values

dF (v)	CL (SL) One-sided	CL (SL) Two-sided	R^2	Best-fit equation (double natural-logarithm transformation: lnln) coefficients I	F_1	F_2	F_3	F_4	F_5	Fitting quality parameter SAR	SSR
2–1000	80 (0.20)	60 (0.40)	0.999949	9.9472937E-01	-1.7648843E-01	2.5064497E-02	3.3241472E-02	-1.0436607E-02	----------------	0.0105	3.453E-06
2–1000	90 (0.10)	80 (0.20)	0.999994	1.6852074E+00	-5.1372638E-01	1.2668298E-01	7.7045230E-02	-3.9283180E-02	4.0107238E-03	0.0093	2.823E-06
2–1000	95 (0.05)	90 (0.10)	0.999999	2.4560595E+00	-1.1327447E+00	4.0703612E-01	8.9347868E-02	-8.0019360E-02	1.2246295E-02	0.0086	2.606E-06
2–1000	97.5 (0.025)	95 (0.05)	0.999999	3.3757524E+00	-2.1607774E+00	1.0167661E+00	-1.6840883E-02	-1.0676906E-01	2.1363795E-02	0.0102	3.697E-06
2–1000	99 (0.01)	98 (0.02)	0.999999	4.9349976E+00	-4.4605325E+00	2.7353719E+00	-5.8881394E-01	-4.9933148E-02	2.7698900E-02	0.0199	1.860E-05
2–1000	99.5 (0.005)	99 (0.01)	0.999999	6.4733597E+00	-7.2463068E+00	5.2481407E+00	-1.7685812E+00	2.3495081E-01	----------------	0.0385	6.817E-05
2–1000	99.9 (0.001)	99.8 (0.002)	1.000000	1.1908885E+01	-1.9789514E+01	1.9327628E+01	-1.0388779E+01	3.0486236E+00	-3.8180629E-01	0.0391	7.859E-05

A total of 42 critical values for degrees of freedom from 1 to 1000 (for a given CL) have been tabulated in different books (*e.g.*, Verma, 2005); however, the double natural-logarithm transformation renders the value of ln(ln(v)) for v=1 as indeterminate. Thus, a total of 41 critical values were used for each CL regressions for v =2–1000.

v: degree of freedom; CL: confidence level; SL: significance level; R^2: coefficient of multiple determination (squared correlation coefficient for a polynomial regression); double natural-logarithm function is lnln – ln(ln(v)); $SAR = \sum Abs(cv_{calc}\text{-}cv_{table})$: sum of absolute values of 41 residuals corresponding to v = 2–1000, *i.e.*, sum of the absolute difference between the calculated critical value (cv_{calc}) from the best-fit equation for a given confidence level and the tabulated critical value (cv_{table}) for v = 2–30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 200, 500, and 1000 (a total of 41 values); similarly, $SSR = \sum(cv_{calc}\text{-}cv_{table})^2$: sum of 41 squared residuals corresponding to v= 2–1000 as above.

Note that for regression parameters with the final term identified as ----------------, the best-fitted equation will be of one order less than the above 5th order equation (see equation 1 in the text).

tions applicable in the range of 1–1000 degrees of freedom for ln-transformation, 2–1000 for lnln-transformation, or 3–1000 for lnlnln-transformation, instead of the tables, can be advantageously proposed as a 21st century innovation for the computer programming of these significance tests. This computer programming work is currently under progress.

APPLICATIONS IN SCIENCE AND ENGINEERING

I first suggest a number of literature references from different areas of science and engineering, which deal with the kind of research for which the new critical value equations will be useful. Then I provide a few actual application examples from a reference material (RM) in geochemistry.

The new equations presented in this work will be useful for the statistical analysis of data in several different fields. A few of them are: agricultural and food science and technology (Pellegrini *et al.*, 2003; Sauvage *et al.*, 2007); biotechnology (Gonzalez *et al.*, 2002); clay mineralogy of sediment cores (Pandarinath, in press); energy and fuels (Bansal *et al.*, 2008); environmental science and technology (Wang *et al.*, 2006); fluid geothermometry (Palabiyik and Serpen, 2008; Díaz-González *et al.*, 2008); instrumentation (Goodman, 1998); medical science and technology (Zacheis *et al.*, 1999; Cooper *et al.*, 2006); proteomic research (Maurer *et al.*, 2005; Verhoeckx *et al.*, 2005; Xia *et al.*, 2006; Verma and Quiroz-Ruiz, 2008); water-rock interaction (Verma *et al.*, 2005); and zoology (Harcourt *et al.*, 2005).

The quality control (assurance and assessment programs) using inter-laboratory data on RM is another important research area where these new critical value equations will be of much use. To name a few of these research areas on the study of RMs, they are: biology and biomedicine (Ihnat, 2000; Patriarca *et al.*, 2005), cement industry (Sieber *et al.*, 2002), environmental and pollution Research (Dybczyński *et al.*, 1998; Gill *et al.*, 2004), food science and technology (Langton *et al.*, 2002; Morabito *et al.*, 2004), organochlorinated compounds and petroleum hydrocarbons in sediments (Villeneuve *et al.*, 2002, 2004); rock chemistry (Verma, 1998; Marroquín-Guerra *et al.*, in press), and water research (Holcombe *et al.*, 2004; Verma, 2004).

The example cases were chosen from the RM database recently used to evaluate the performance of single discordant-outlier tests (Verma *et al.*, 2009). It is important to remind that the Student t and Fisher F-ratio tests are sensitive to the presence of discordant outliers (Jensen *et al.*, 2000). For the ANOVA test also, the data to be examined should ideally be free from discordant outliers. Therefore, in order to correctly apply these tests, the individual datasets should first be processed for the possible presence of such outliers using appropriate discordancy tests (Barnett and Lewis, 1994) along with the new, precise, and accurate criti-

Table 2. The best regression double natural-logarithm transformed model for the interpolation of the *vertical* degrees of freedom (VdF – v_2) – 38 (out of a total of 39) critical values for each regression, with the horizontal degree of freedom (HdF – v_1) maintained constant for a given set – Fisher F for 99% confidence level.

HdF (v_1)	VdF (v_2)	R^2	Best-fit equation (double natural logarithm transformation: lnln) coefficients for 99% confidence level							Fitting quality parameter	
			I	F_1	F_2	F_3	F_4	F_5	F_6	*SAR*	*SSR*
1	2–1000	1.000000	4.2026579E+01	-9.4626623E+01	1.2083250E+02	-9.4316650E+01	4.5921907E+01	-1.2682276E+01	1.5029694E+00	0.0713	2.336E-04
2	2–1000	1.000000	3.8901126E+01	-9.7448059E+01	1.3282906E+02	-1.0996596E+02	5.6182988E+01	-1.6140695E+01	1.9789521E+00	0.0899	3.063E-04
3	2–1000	1.000000	3.7602911E+01	-9.8506802E+01	1.3772734E+02	-1.1664282E+02	6.0744393E+01	-1.7733543E+01	2.2040277E+00	0.0602	1.780E-04
4	2–1000	1.000000	3.6887674E+01	-9.9070385E+01	1.4039128E+02	-1.2029581E+02	6.3229622E+01	-1.8594674E+01	2.3251695E+00	0.0777	2.474E-04
5	2–1000	1.000000	3.6437368E+01	-9.9434118E+01	1.4205563E+02	-1.2257012E+02	6.4773288E+01	-1.9124724E+01	2.3982209E+00	0.0709	2.200E-04
6	2–1000	1.000000	3.6124079E+01	-9.9682248E+01	1.4319527E+02	-1.2413762E+02	6.5846033E+01	-1.9497770E+01	2.4505499E+00	0.0809	2.574E-04
7	2–1000	1.000000	3.5898902E+01	-9.9887963E+01	1.4404565E+02	-1.2520492E+02	6.6485126E+01	-1.9687235E+01	2.4732884E+00	0.0824	2.787E-04
8	2–1000	1.000000	3.5738681E+01	-1.0006334E+02	1.4459615E+02	-1.2576285E+02	6.6729689E+01	-1.9725961E+01	2.4727506E+00	0.0665	1.814E-04
9	2–1000	1.000000	3.5578508E+01	-1.0010225E+02	1.4519336E+02	-1.2700519E+02	6.7888323E+01	-2.0236659E+01	2.5586609E+00	0.0742	2.397E-04
10	2–1000	1.000000	3.5467711E+01	-1.0021280E+02	1.4560466E+02	-1.2744592E+02	6.8064512E+01	-2.0245548E+01	2.5522051E+00	0.0781	2.404E-04
11	2–1000	1.000000	3.5368148E+01	-1.0026519E+02	1.4596124E+02	-1.2807727E+02	6.8609903E+01	-2.0478679E+01	2.5911849E+00	0.0876	2.863E-04
12	2–1000	1.000000	3.5279949E+01	-1.0030905E+02	1.4631369E+02	-1.2863235E+02	6.8980355E+01	-2.0587404E+01	2.6018935E+00	0.0873	3.079E-04
15	2–1000	1.000000	3.5109760E+01	-1.0046604E+02	1.4690356E+02	-1.2934515E+02	6.9394360E+01	-2.0711724E+01	2.6182212E+00	0.0851	2.609E-04
20	2–1000	1.000000	3.4938483E+01	-1.0060010E+02	1.4752932E+02	-1.3026566E+02	7.0021574E+01	-2.0914434E+01	2.6430691E+00	0.0883	3.026E-04
25	2–1000	1.000000	3.4860767E+01	-1.0075951E+02	1.4777142E+02	-1.3028133E+02	6.9844751E+01	-2.0801345E+01	2.6219973E+00	0.0857	2.915E-04
30	2–1000	1.000000	3.4783282E+01	-1.0079756E+02	1.4804105E+02	-1.3070761E+02	7.0148883E+01	-2.0913445E+01	2.6398910E+00	0.0748	2.460E-04
40	2–1000	1.000000	3.4695047E+01	-1.0085416E+02	1.4836400E+02	-1.3123967E+02	7.0539037E+01	-2.1049541E+01	2.6580569E+00	0.0967	3.540E-04
50	2–1000	1.000000	3.4642387E+01	-1.0094628E+02	1.4859055E+02	-1.3130486E+02	7.0341510E+01	-2.0881050E+01	2.6196484E+00	0.0883	3.039E-04
100	2–1000	1.000000	3.4541762E+01	-1.0105219E+02	1.4891250E+02	-1.3173167E+02	7.0683448E+01	-2.1053069E+01	2.6559845E+00	0.0887	3.267E-04
1000	2–1000	1.000000	3.4445104E+01	-1.0112815E+02	1.4926391E+02	-1.3234018E+02	7.1230587E+01	-2.1319104E+01	2.7066799E+00	0.0556	1.568E-04

v_1: horizontal degree of freedom (HdF); v_2: vertical degree of freedom (VdF); R^2 and lnln are the same as in Table 1. *SAR*: sum of absolute values of 38 residuals corresponding to v_2 = 2–1000; *i.e.*, sum of the absolute difference between the calculated critical value from the best-fit equation (2) for 99% confidence level (cv_{calc}) and the tabulated critical value (cv_{table}) for v_2 =2–30, 35, 40, 50, 60, 80, 100, 200, 500, and 1000; similarly, *SSR*: sum of 38 squared residuals corresponding to v_2 = 2–1000.

Table 3. The best double natural-logarithm transformed regression model for the interpolation of the *horizontal* degrees of freedom (HdF v_1) – 19 (out of a total of 20) critical values for each regression, with the vertical degree of freedom (VdF v_2) maintained constant for a given set – Fisher F for 99% confidence level.

dFV (v_2)		R^2	Best-fit equation (double natural logarithm transformation: lnln) coefficients for 95% confidence level						Fitting Quality parameter	
			I	F_1	F_2	F_3	F_4	F_5	***SAR***	***SSR***
*1	2–1000	0.996713	4.0314820E+02	1.8375277E+02	-6.3150695E+01	1.0854863E+01	-7.0265963E-01	----------------	38.62	1.89E+02
*2	2–1000	0.999518	9.9131397E+01	3.7663870E-01	6.6136753E-03	-1.1541409E-01	3.2894339E-02	----------------	0.0430	1.37E-04
3	2–1000	0.999961	2.9756741E+01	-3.1023100E+00	-4.6582261E-01	4.1560437E-01	4.3172308E-01	-1.8192871E-01	0.1043	1.04E-03
4	2–1000	0.999990	1.6976329E+01	-2.9783196E+00	-4.2119570E-01	3.9600706E-01	3.7688624E-01	-1.5889941E-01	0.0556	2.52E-04
5	2–1000	0.999990	1.2325498E+01	-2.7743222E+00	-4.3138316E-01	4.2651483E-01	3.0691071E-01	-1.3661980E-01	0.0465	2.12E-04
6	2–1000	0.999990	1.0030138E+01	-2.6072041E+00	-4.1490859E-01	3.4842804E-01	3.3625343E-01	-1.3938720E-01	0.0459	1.85E-04
7	2–1000	0.999992	8.6928867E+00	-2.5146543E+00	-3.9030701E-01	3.6734185E-01	2.7128420E-01	-1.1690439E-01	0.0408	1.50E-04
8	2–1000	0.999988	7.8238477E+00	-2.4220376E+00	-3.8683723E-01	3.2370025E-01	2.9181596E-01	-1.2003305E-01	0.0511	2.07E-04
9	2–1000	0.999991	7.2152650E+00	-2.3510441E+00	-3.6976380E-01	2.7859308E-01	3.0405081E-01	-1.1956486E-01	0.0448	1.54E-04
10	2–1000	0.999989	6.7696417E+00	-2.2981440E+00	-3.5027858E-01	2.3545589E-01	3.1898984E-01	-1.2054595E-01	0.0488	1.76E-04
11	2–1000	0.999995	6.4345100E+00	-2.2498877E+00	-3.5773564E-01	1.7769498E-01	3.7501671E-01	-1.3555035E-01	0.0292	7.91E-05
12	2–1000	0.999995	6.1597635E+00	-2.2164383E+00	-3.1894534E-01	1.3691119E-01	3.7450194E-01	-1.3083733E-01	0.0260	7.15E-05
13	2–1000	0.999990	5.9518332E+00	-2.1948294E+00	-3.7678053E-01	2.4400108E-01	2.9884039E-01	-1.1325453E-01	0.0415	1.52E-04
14	2–1000	0.999988	5.7684929E+00	-2.1598104E+00	-3.5419027E-01	1.8989168E-01	3.2387727E-01	-1.1649540E-01	0.0482	1.75E-04
15	2–1000	0.999991	5.6232239E+00	-2.1426425E+00	-3.5429461E-01	1.6227553E-01	3.4947114E-01	-1.2334749E-01	0.0389	1.29E-04
16	2–1000	0.999993	5.4915662E+00	-2.1213006E+00	-3.1298924E-01	9.4737688E-02	3.7870066E-01	-1.2721776E-01	0.0356	1.05E-04
17	2–1000	0.999993	5.3813637E+00	-2.0971502E+00	-3.2194040E-01	9.2200964E-02	3.7746543E-01	-1.2601560E-01	0.0391	1.04E-04
18	2–1000	0.999990	5.2893284E+00	-2.0878916E+00	-3.3641621E-01	1.2956452E-01	3.4519549E-01	-1.1752266E-01	0.0401	1.37E-04
19	2–1000	0.999993	5.2075280E+00	-2.0790950E+00	-3.1770469E-01	8.7841273E-02	3.7079039E-01	-1.2275590E-01	0.0352	9.57E-05
20	2–1000	0.999990	5.1357518E+00	-2.0707045E+00	-3.3608680E-01	1.2736817E-01	3.4127131E-01	-1.1603619E-01	0.0449	1.39E-04
21	2–1000	0.999991	5.0640679E+00	-2.0448522E+00	-3.0172737E-01	2.4807259E-02	4.1382978E-01	-1.3230738E-01	0.0374	1.26E-04
22	2–1000	0.999991	5.0140116E+00	-2.0551043E+00	-3.3914883E-01	1.5506657E-01	3.0085244E-01	-1.0280769E-01	0.0402	1.20E-04
23	2–1000	0.999992	4.9536716E+00	-2.0356702E+00	-3.1287114E-01	9.3136073E-02	3.3864279E-01	-1.1043997E-01	0.0345	1.06E-04
24	2–1000	0.999993	4.9121230E+00	-2.0202982E+00	-3.4055875E-01	8.5313994E-02	3.6615649E-01	-1.2015596E-01	0.0351	9.58E-05
25	2–1000	0.999989	4.8721566E+00	-2.0308848E+00	-3.4197628E-01	1.3688738E-01	3.1394408E-01	-1.0598398E-01	0.0416	1.48E-04
26	2–1000	0.999993	4.8325012E+00	-2.0266315E+00	-3.3764551E-01	1.2812038E-01	3.1800638E-01	-1.0666589E-01	0.0324	9.01E-05
27	2–1000	0.999990	4.7914178E+00	-2.0105866E+00	-3.1247190E-01	6.8787495E-02	3.5322675E-01	-1.1322390E-01	0.0372	1.33E-04
28	2–1000	0.999988	4.7579617E+00	-2.0062956E+00	-3.2816427E-01	1.0893686E-01	3.1999846E-01	-1.0477917E-01	0.0464	1.57E-04
29	2–1000	0.999989	4.7289819E+00	-2.0036985E+00	-3.3092416E-01	1.0760295E-01	3.2365984E-01	-1.0616761E-01	0.0432	1.56E-04
30	2–1000	0.999991	4.7021161E+00	-2.0076611E+00	-3.3950832E-01	1.5973493E-01	2.7194582E-01	-9.2009493E-02	0.0397	1.24E-04
35	2–1000	0.999992	4.5881799E+00	-1.9857156E+00	-3.3757353E-01	1.3005469E-01	2.9274676E-01	-9.6890382E-02	0.0368	1.09E-04
40	2–1000	0.999992	4.4973429E+00	-1.9665045E+00	-3.0694926E-01	6.7350575E-02	3.2469249E-01	-1.0184803E-01	0.0364	1.08E-04
50	2–1000	0.999992	4.3885903E+00	-1.9621807E+00	-3.3037042E-01	1.6573380E-01	2.2762138E-01	-7.5464559E-02	0.0353	1.13E-04
60	2–1000	0.999996	4.3160874E+00	-1.9508948E+00	-3.4287507E-01	1.8776369E-01	2.0721878E-01	-7.0147551E-02	0.0246	5.28E-05
80	2–1000	0.999988	4.2223671E+00	-1.9282050E+00	-3.3158137E-01	1.7038076E-01	1.9651644E-01	-6.3748812E-02	0.0469	1.56E-04
100	2–1000	0.999990	4.1626110E+00	-1.9089098E+00	-3.1309893E-01	1.0607497E-01	2.4640856E-01	-7.6307245E-02	0.0410	1.29E-04
200	2–1000	0.999993	4.0599847E+00	-1.9015159E+00	-3.1469211E-01	1.6192612E-01	1.7176712E-01	-5.4693783E-02	0.0346	9.24E-05
500	2–1000	0.999987	4.0012380E+00	-1.8913868E+00	-3.0759045E-01	1.4068633E-01	1.8476742E-01	-5.8985288E-02	0.0499	1.79E-04
1000	2–1000	0.999985	2.0510780E+00	-1.6515341E+00	-1.9189245E-01	6.2471180E-01	4.6880445E-01	1.0119271E-01	0.0404	1.22E-04

v_1, v_2, R^2, lnln, and ---------------- are the same as in Tables 1-2. *SAR*: sum of absolute values of 19 residuals corresponding to v_1 = 2–1000, for v_1 = 2–12, 15, 20, 25, 30, 40, 50, 100, and 1000 (a total of 19 values); similarly, *SSR*: sum of 19 squared residuals corresponding to v_1 = 2–1000.

* For v_2=1, the regression equation reported is a double natural-logarithm transformed 4th order equation, and not 5th order one as in all other cases except for v_2=2, for which it is a single natural-logarithm transformed 4th order equation.

Table 4. Examples of the best "second round" regression models for the *vertical* degrees of freedom (VdF – v_2) – 37 (out of a total of 39) critical values for each regression, with the horizontal degree of freedom (HdF – v_1) maintained constant for a given set – Fisher F for 99% confidence level

HdF (v_1)	VdF (v_2)	R^2	Best-fit equation (triple natural logarithm transformation: lnlnln) coefficients for 99% confidence level							Fitting quality parameter	
			I	F_1	F_2	F_3	F_4	F_5	F_6	*SAR*	*SSR*
13	3–1000	1.000000	3.5927434E+00	-4.9348405E+00	4.8910020E+00	-4.6191297E-01	-1.0463481E+00	6.5853174E-02	9.0642396E-02	---	---
14	3–1000	1.000000	3.5451983E+00	-4.9302421E+00	4.8787045E+00	-4.6504805E-01	-1.0379426E+00	7.1288708E-02	9.1585434E-02	---	---
16	3–1000	1.000000	3.4664943E+00	-4.9244003E+00	4.8564254E+00	-4.7099717E-01	-1.0220156E+00	8.2600548E-02	9.3752290E-02	---	---
17	3–1000	1.000000	3.4335309E+00	-4.9226768E+00	4.8462457E+00	-4.7387233E-01	-1.0144697E+00	8.8375265E-02	9.4933093E-02	---	---
18	3–1000	1.000000	3.4039406E+00	-4.9215291E+00	4.8366080E+00	-4.7670162E-01	-1.0071877E+00	9.4176035E-02	9.6156470E-02	---	---
19	3–1000	1.000000	3.3772251E+00	-4.9208393E+00	4.8274556E+00	-4.7949226E-01	-1.0001591E+00	9.9972614E-02	9.7409586E-02	---	---
21	3–1000	1.000000	3.3308754E+00	-4.9204872E+00	4.8104260E+00	-4.8497098E-01	-9.8682050E-01	1.1146254E-01	9.9965663E-02	---	---
22	3–1000	1.000000	3.3106365E+00	-4.9206967E+00	4.8024750E+00	-4.8766164E-01	-9.8049032E-01	1.1712255E-01	1.0125363E-01	---	---
23	3–1000	1.000000	3.2920352E+00	-4.9210992E+00	4.7948591E+00	-4.9032015E-01	-9.7437313E-01	1.2270990E-01	1.0254063E-01	---	---
24	3–1000	1.000000	3.2748791E+00	-4.9216582E+00	4.7875524E+00	-4.9294627E-01	-9.6845949E-01	1.2821617E-01	1.0382241E-01	---	---
26	3–1000	1.000000	3.2442738E+00	-4.9231335E+00	4.7737786E+00	-4.9809988E-01	-9.5720690E-01	1.3896243E-01	1.0635740E-01	---	---
27	3–1000	1.000000	3.2305657E+00	-4.9240062E+00	4.7672734E+00	-5.0062662E-01	-9.5185091E-01	1.4419492E-01	1.0760581E-01	---	---
28	3–1000	1.000000	3.2177774E+00	-4.9249461E+00	4.7610004E+00	-5.0311962E-01	-9.4666440E-01	1.4933075E-01	1.0883915E-01	---	---
29	3–1000	1.000000	3.2058192E+00	-4.9259398E+00	4.7549451E+00	-5.0557869E-01	-9.4163981E-01	1.5436890E-01	1.1005613E-01	---	---

v_1, v_2, R^2, *SAR*, and *SSR* are the same as in Table 2, except that the latter two parameters are for 37 residuals corresponding to v_2 = 3–1000. The triple natural-logarithm transformation lnlnln= $\ln(\ln(\ln(v^2)))$. Note that it is not possible to estimate the fitting quality parameters *SAR* and *SSR*, because these critical values are totally absent from the original tables; the columns are included to highlight this fact.

Table 5. Examples of the best "second round" regression models for the *horizontal* degrees of freedom (VdF v_1) – 19 (out of a total of 20) critical values for each regression, with the vertical degree of freedom (HdF v_2) maintained constant for a given set – Fisher F for 99% confidence level.

VdF (v_2)	HdF (v_1)	R^2	Best-fit equation (double natural logarithm transformation: lnln) coefficients for 95% confidence level						Fitting quality parameter	
			I	F_1	F_2	F_3	F_4	F_5	*SAR*	*SSR*
31	2–1000	0.999999	4.6727856E+00	-1.9966807E+00	-3.2674642E-01	1.0956190E-01	3.1211521E-01	-1.0187491E-01	---	---
32	2–1000	0.999999	4.6484436E+00	-1.9926255E+00	-3.2650769E-01	1.0968084E-01	3.0942477E-01	-1.0081937E-01	---	---
33	2–1000	0.999999	4.6257490E+00	-1.9888293E+00	-3.2629268E-01	1.0989445E-01	3.0677301E-01	-9.9792360E-02	---	---
34	2–1000	0.999999	4.6045411E+00	-1.9852688E+00	-3.2609857E-01	1.1018939E-01	3.0416286E-01	-9.8793274E-02	---	---
36	2–1000	0.999999	4.5660398E+00	-1.9787754E+00	-3.2576369E-01	1.1097886E-01	2.9907544E-01	-9.6876224E-02	---	---
37	2–1000	0.999999	4.5485136E+00	-1.9758080E+00	-3.2561898E-01	1.1145481E-01	2.9660084E-01	-9.5956841E-02	---	---
38	2–1000	0.999999	4.5320041E+00	-1.9730069E+00	-3.2548721E-01	1.1197453E-01	2.9417334E-01	-9.5062572E-02	---	---
39	2–1000	0.999999	4.5164253E+00	-1.9703592E+00	-3.2536696E-01	1.1253149E-01	2.9179328E-01	-9.4192670E-02	---	---
41	2–1000	0.999999	4.4877618E+00	-1.9654784E+00	-3.2515625E-01	1.1373505E-01	2.8717557E-01	-9.2522993E-02	---	---
42	2–1000	0.999999	4.4745473E+00	-1.9632253E+00	-3.2506374E-01	1.1437232E-01	2.8493745E-01	-9.1721750E-02	---	---
43	2–1000	0.999999	4.4620017E+00	-1.9610853E+00	-3.2497860E-01	1.1502799E-01	2.8274589E-01	-9.0941947E-02	---	---
44	2–1000	0.999999	4.4500755E+00	-1.9590507E+00	-3.2490009E-01	1.1569870E-01	2.8060031E-01	-9.0182885E-02	---	---
45	2–1000	0.999999	4.4387234E+00	-1.9571143E+00	-3.2482754E-01	1.1638153E-01	2.7850004E-01	-8.9443882E-02	---	---
46	2–1000	0.999999	4.4279047E+00	-1.9552696E+00	-3.2476034E-01	1.1707388E-01	2.7644433E-01	-8.8724278E-02	---	---
47	2–1000	0.999999	4.4175824E+00	-1.9535107E+00	-3.2469796E-01	1.1777349E-01	2.7443236E-01	-8.8023430E-02	---	---
48	2–1000	0.999999	4.4077226E+00	-1.9518322E+00	-3.2463991E-01	1.1847838E-01	2.7246329E-01	-8.7340716E-02	---	---
49	2–1000	0.999999	4.3982946E+00	-1.9502291E+00	-3.2458577E-01	1.1918677E-01	2.7053625E-01	-8.6675537E-02	---	---

For abbreviations and explanation see Tables 1-4, in particular Table 3.

Table 6. Synthesis of the major element data in diabase W-1 (RM issued by the U. S. Geological Survey, U.S.A.) from different analytical method groups.

Element/ Oxide	**Method Group ***	**Initial Statistics** n_{in}	$\bar{x}_{in}$	s_{in}	***Ot***	**Final Statistics** n_{out}	$\bar{x}$	s	**Significance test applied** (all group-combination **recommended**)
SiO_2	Gr1	79	52.363	0.312	8	71	52.446	0.184	ANOVA: TS_{sample} = 4.8177; cv_{calc}=[v_1=6, v_2=184]= 2.902 (**no**)
	Gr2	14	52.31	1.22	5	9	52.663	0.080	
	Gr3	27	52.16	1.46	4	23	52.664	0.558	
	Gr4	15	52.99	1.95	5	10	52.326	0.358	
	Gr5	11	52.51	0.75	3	8	52.665	0.214	
	Gr6 **	4	51.88	1.23	0	4	51.88	1.23	
	Gr8	34	52.538	0.240	4	30	52.474	0.153	
	Gr1, Gr4, Gr8 (all)	128	52.48	0.73					ANOVA: TS_{sample}= 2.1310; cv_{calc}=[v_1=2, v_2=108]= 4.812 (**yes**)
		111	52.443	0.199	5	**106**	**52.458**	**0.162**	
	Gr2, Gr3, Gr5 (all)	52	52.28	1.26					ANOVA: TS_{sample}= 0.00004; cv_{calc}=[v_1=2, v_2=37]= 5.229 (**yes**)
		40	52.664	0.430	5	**35**	**52.585**	**0.281**	
TiO_2	Gr1	12	1.073	0.052	2	10	1.072	0.092	ANOVA: TS_{sample}= 0.9768; cv_{calc}=[v_1=6, v_2=185]= 2.901 (**yes**)
	Gr2	34	1.062	0.052	5	29	1.0922	0.0220	
	Gr3	43	1.091	0.167	5	38	1.0744	0.0227	
	Gr4	14	1.031	0.089	0	14	1.059	0.097	
	Gr5	7	1.058	0.209	0	7	1.031	0.089	
	Gr6	66	1.072	0.160	12	54	1.058	0.209	
	Gr8	45	1.026	0.212	5	40	1.0774	0.0294	
	Gr1-Gr8(all)	221	1.062	0.157					
	Gr1-Gr8	192	1.069	0.077	17	**175**	**1.0736**	**0.0444**	
Al_2O_3	Gr1	83	15.10	0.57	11	72	15.018	0.212	ANOVA: TS_{sample}= 0.4048; cv_{calc}=[v_1=5, v_2=169]= 3.130 (**yes**)
	Gr2	23	14.985	0.337	4	19	14.919	0.181	
	Gr3	26	15.057	0.327	4	22	15.029	0.165	
	Gr4	20	15.23	0.84	5	15	14.990	0.230	
	Gr5	15	14.991	0.76	0	15	14.99	0.76	
	Gr8	32	14.989	0.197	0	32	14.989	0.197	
	Gr1-Gr8(all)	199	15.07	0.53					
	Gr1-Gr8	175	15.001	0.291	8	**167**	**15.003**	**0.201**	
$Fe_2O_3^t$	Gr1	53	11.19	0.50	7	46	11.196	0.206	ANOVA: TS_{sample}= 20.922; cv_{calc}=[v_1=6, v_2=194]= 2.896 (**no**)
	Gr2	30	10.991	0.286	4	26	11.085	0.138	
	Gr3	31	11.113	0.271	0	31	11.113	0.271	
	Gr6 **	3	13.00	1.54	0	3	13.00	1.54	
	Gr8	59	11.27	0.66	9	50	11.122	0.122	
	Gr1-Gr4, Gr8	168	11.134	0.204	6	**162**	**11.114**	**0.161**	ANOVA: TS_{sample}= 2.5601; cv_{calc}=[v_1=4, v_2=163]= 3.437 (**yes**)

cal values (Verma and Quiroz-Ruiz, 2006a, 2006b, 2008; Verma *et al.*, 2008).

A large number of examples exist in this extensive RM database that fall in the category of actual gaps in the critical value tables, for example, for the Student t the dF or v as 30<v<1000, but distinct from v = 30(5)50(10)100(100)200(300)500(500)1000, *i.e.*, different from the tabulated v of 35, 40, 45, 50, 60, 70, 80, 90, 100, 200, 500, or 1,000. Similarly, for the Fisher F tables, such gaps exist for horizontal dF (HdF) v_1>12, i.e., v_1 different from 12(3)15(5)30(10)50(50)100(900)1000 and for vertical dF (VdF) v_2>30, i.e., v_2 different from 30(5)40(10)60(20)100(100)200(300)500(500)1000.

The examples (Table 6) include the major element data in the RM diabase W-1 from the United States Geological Survey (U.S.A.) compiled by Verma *et al*. (2009) that require the use of newly interpolated critical values. The extensive footnote of Table 6 provides more information on the application of the ANOVA, F-ratio and Student t tests.

For TiO_2, Al_2O_3, MgO, CaO, K_2O, and P_2O_5, the ANOVA test and for H_2O^+, and H_2O^-, the F-ratio and Student t tests showed that the data from the different analytical method groups can be combined into a single group and an overall mean and standard deviation as well as confidence limits of the mean can be calculated. For the remaining

Table 6 (Continued). Synthesis of the major element data in diabase W-1 (RM issued by the U. S. Geological Survey, U.S.A.) from different analytical method groups.

Element/ Oxide	**Method Group ***	**Initial Statistics** n_{in}	$\bar{x}_{in}$	s_{in}	**Ot**	**Final Statistics** n_{out}	$\bar{x}$	s	**Significance test applied** (all group-combination **recommended**)
MnO	Gr1	38	0.181	0.067	5	33	0.1676	0.0261	ANOVA: TS_{sample}= 3.2224; cv_{calc}=[v_1=6, v_2=207]= 2.888 (**no**)
	Gr2	27	0.1660	0.0142	4	23	0.1705	0.0071	
	Gr3	32	0.1713	0.0125	2	30	0.1692	0.0095	
	Gr4	50	0.1768	0.0250	4	46	0.1716	0.0179	
	Gr5	24	0.1682	0.0127	0	24	0.1682	0.0127	
	Gr6 **	6	0.1844	0.0285	0	6	0.1844	0.0285	
	Gr8	58	0.1647	0.0296	6	52	0.1606	0.0132	
	Gr1-Gr5, Gr8	208	0.1670	0.0171	5	**203**	**0.1677**	**0.0148**	ANOVA: TS_{sample}= 2.7588; cv_{calc}=[v_1=5, v_2=202]= 3.108 (**yes**)
MgO	Gr1	81	6.52	0.54	11	70	6.615	0.131	ANOVA: TS_{sample}= 12.072; cv_{calc}=[v_1=6, v_2=168]= 2.914 (**no**)
	Gr2	34	6.662	0.172	4	30	6.617	0.115	
	Gr3	24	6.570	0.249	4	20	6.655	0.138	
	Gr4	22	6.81	0.77	6	16	6.6134	0.0442	
	Gr5 **	8	6.20	0.53	0	8	6.20	0.53	
	Gr6 **	3	7.89	2.26	0	3	7.89	2.26	
	Gr8	28	6.631	0.131	0	28	6.631	0.131	
	Gr1-Gr4, Gr8	164	6.621	0.117	0	**164**	**6.621**	**0.117**	ANOVA: TS_{sample}= 0.4861; cv_{calc}=[v_1=4, v_2=159]= 3.440 (**yes**)
CaO	Gr1	83	10.962	0.194	5	78	10.943	0.140	ANOVA: TS_{sample} = 2.8114; cv_{calc}=[v_1=6, v_2=193]= 2.896 (**yes**)
	Gr2	33	10.899	0.196	4	29	10.845	0.128	
	Gr3	34	10.963	0.342	5	29	10.892	0.198	
	Gr4	21	11.11	0.49	0	21	11.11	0.49	
	Gr5	11	10.93	0.45	0	11	10.93	0.45	
	Gr6	4	11.005	0.347	0	4	11.005	0.347	
	Gr8	28	10.901	0.184	0	28	10.901	0.184	
	Gr1-Gr8(all)	214	10.958	0.283					
	Gr1-Gr8	200	10.933	0.248	10	**190**	**10.916**	**0.171**	
Na_2O	Gr1 **	70	2.071	0.201	0	70	2.071	0.201	ANOVA: TS_{sample}= 4.2475; cv_{calc}=[v_1=6, v_2=188]= 2.899 (**no**)
	Gr2	67	2.235	0.497	5	62	2.168	0.058	
	Gr3	16	2.210	0.143	0	16	2.210	0.143	
	Gr4	12	2.115	0.100	0	12	2.115	0.100	
	Gr5	27	2.162	0.124	0	27	2.162	0.124	
	Gr6 **	3	2.260	0.131	0	3	2.260	0.131	
	Gr8	5	2.173	0.071	0	5	2.173	0.071	
	Gr2-Gr5, Gr8	122	2.167	0.095	0	**122**	**2.167**	**0.095**	ANOVA: TS_{sample}= 1.7602; cv_{calc}=[v_1=4, v_2=117]= 3.489. (**yes**)

major elements or oxides (SiO_2, $Fe_2O_3^t$, MnO and Na_2O), statistically significant differences were observed among the method groups, and the combination of all methods into a single group was therefore not recommended. For the latter cases, the overall statistical parameters were calculated for only those method groups that showed no significant differences among them.

For these significance tests (ANOVA, F-ratio, and Student t), the new equations provided precise interpolated critical values as documented in the earlier section. If we were to calculate the 95% or 99% confidence limits of the mean (not included in Table 6), we would also need precise critical values for the Student t test corresponding to the appropriate degrees of freedom (v).

CONCLUSIONS

The criteria of the multiple-correlation coefficient (R^2) and the interpolation residuals (*SAR* and *SSR*) clearly suggest that simple polynomial regressions are not appropriate for the interpolation of the Student t and Fisher F critical values. The ln-transformation is a required operation to achieve better regression models. The ln-, lnln-, and lnlnln-transfor-

Table 6 (Continued). Synthesis of the major element data in diabase W-1 (RM issued by the U. S. Geological Survey, U.S.A.) from different analytical method groups.

Element/ Oxide	Method Group *	Initial Statistics			*Ot*	Final Statistics			Significance test applied (all group-combination **recommended**)
		n_{in}	$\bar{x}_{in}$	s_{in}		n_{out}	$\bar{x}$	s	
K_2O	Gr1	70	0.686	0.227	8	62	0.647	0.054	ANOVA: TS_{sample}= 2.4690; cv_{calc}=[v_1=6, v_2=186]= 2.901 (**yes**)
	Gr2	64	0.6432	0.0387	2	62	0.6465	0.0346	
	Gr3	35	0.654	0.050	4	31	0.6406	0.0288	
	Gr4	15	0.629	0.187	4	11	0.6398	0.0257	
	Gr5	18	0.658	0.066	4	14	0.6298	0.0266	
	Gr6	8	0.575	0.165	0	8	0.575	0.165	
	Gr8	5	0.624	0.106	0	5	0.624	0.106	
	Gr1-Gr8(all)	215	0.656	0.147					
	Gr1-Gr8	193	0.641	0.054	9	**184**	**0.6413**	**0.0348**	
P_2O_5	Gr1	39	0.1424	0.0403	0	39	0.1424	0.0403	ANOVA: TS_{sample}=0.8809; cv_{calc}=[v_1=5, v_2=126]= 3.173 (**yes**)
	Gr3	20	0.1423	0.0360	0	20	0.1423	0.0360	
	Gr4	8	0.1223	0.0125	0	8	0.1223	0.0125	
	Gr5	9	0.1321	0.0097	0	9	0.1321	0.0097	
	Gr6	4	0.1340	0.0343	0	4	0.1340	0.0343	
	Gr8	63	0.158	0.085	11	52	0.1383	0.0111	
	Gr1-Gr8(all)	143	0.147	0.063					
	Gr1-Gr8	132	0.1387	0.0278	8	**124**	**0.1363**	**0.0205**	
H_2O^+	Gr1	45	0.586	0.200	5	40	0.574	0.124	F-ratio: TS_{sample}= 1.5250; cv_{calc} (v=68) = 6.9988 (**yes**)
	Gr8	30	0.486	0.154	0	30	0.486	0.154	Student t: TS_{sample}= 2.5443; cv_{calc} (v=68) = 2.6498 (**yes**)
	Gr1, Gr8	70	0.536	0.143	0	**70**	**0.536**	**0.143**	
H_2O^-	Gr1	69	0.148	0.060	4	65	0.140	0.048	F-ratio: TS_{sample}= 1.1380; cv_{calc} (v=69) = 6.9988 (**yes**)
	Gr8	6	0.1283	0.0454	0	6	0.1283	0.0454	Student t: TS_{sample}= 0.55548; cv_{calc} (v=69) = 2.6487 (**yes**)
	Gr1, Gr8	71	0.139	0.048	0	**71**	**0.139**	**0.048**	

***** Method Group refers to the analytical methods as follows (Velasco-Tapia *et al.*, 2001): Gr1: classical methods; Gr2: atomic absorption methods; Gr3: x-ray fluorescence methods; Gr4: emission spectrometry methods; Gr5: nuclear methods; Gr6: mass spectrometry methods; Gr7: chromatography methods; and Gr8: miscellaneous methods.

****** The method group for an element identified by these two asterisks gave significantly different results than all other groups for the determination of this particular element.

The initial and final statistics, respectively, refer to the data before and after the application of discordancy tests using new critical values (Verma and Quiroz-Ruiz, 2006a, 2006b, 2008; Verma *et al.*, 2008). n_{in}: initial number of observations or analytical data by a given method group; $\bar{x}_{in}$: mean of all initial observations or analytical data by a given method group; s_{in}: standard deviation of all initial observations or analytical data by a given method group; *Ot*: number of discordant outliers mean of all initial observations or analytical data by a given method group; n_{out}: final number of observations or analytical data by a given method group after the detection and elimination of discordant outliers; $\bar{x}$: mean of final observations or analytical data by a given method group after the detection and elimination of discordant outliers; s: standard deviation of final observations or analytical data by a given method group after the detection and elimination of discordant outliers. The final statistics for the combined groups is given in bold face.

The significance tests are: ANOVA: analysis of variance for three or more samples, *i.e.*, analytical data for at least three method groups; Student t: t test for two means, *i.e.*, analytical data for two method groups; Fisher F: F-ratio test for two variances; TS_{sample}– test statistic for the samples under analysis of significance test; cv_{calc}: critical value for the degrees of freedom of the samples under analysis of significance test as obtained from the best interpolation equations proposed in this work. All tests were applied at the strict CL of 99%.

The null hypothesis (H_0) is that all samples were drawn from the same or equivalent normal population; the alternate hypothesis (H_1) is that they were not drawn from the same normal population, *i.e.*, at least one of them came from a different normal population. H_0 is said to be valid when $TS_{sample}<cv_{calc}$; else, when $TS_{sample}\geq cv_{calc}$, H_1 is valid.

In terms of interpretation of the analytical data from different groups, this hypothesis test refers to the result for the group-combination **recommended**: as **yes** if $TS_{sample}<cv_{calc}$, or **no** if $TS_{sample}\geq cv_{calc}$.

For the ANOVA or Fisher F-ratio test, the degrees of freedom were: the Horizontal degrees of freedom v_1=k-1 where k is the number of analytical groups (statistical samples) being evaluated; the Vertical degrees of freedom v_2=N-k where N is the total number of observations in the entire dataset, *i.e.*, in all statistical samples.

In this report, one extra digit was maintained as suggested by Bevington and Robinson (2003) and Verma (2005).

mations combined with the polynomial regression models, were shown to perform better than the simple polynomial models. The best interpolation models involved lnln- and lnlnln-transformations prior to the polynomial fits. The use of these new best interpolation equations in spreadsheet calculations or computer programs is recommended for all applications in science and engineering involving these significance tests. Finally, the new interpolated critical values for the Student t test would be useful to calculate more precisely the 95% or 99% confidence limits of the mean.

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